Speed Reducers for Precision Motion Control

HarmonicDrive®
Reducer Catalog

- Differential Gear FD
- Engineering Data
Excellent Technology for Evolving Industries

Harmonic Drive® actuators utilize high-precision, zero-backlash Harmonic Drive® precision gears and play critical roles in robotics, semiconductor manufacturing equipment, factory automation equipment, medical diagnostics and surgical robotics. Additionally, our products are frequently used in mission-critical spaceflight applications which capture the human spirit.

With over 50 years of experience, our expert engineering and production teams continually develop enabling technologies for the evolving motion control market. We are proud of our outstanding engineering capabilities and successful history of providing customer specific solutions to meet their application requirements.

Harmonic Drive LLC continues to develop enabling technologies for the evolving motion control market, which drives the pace of global innovation.

C. Walton Musser
Patented Strain Wave Gearing in 1955
Operating Principle of HarmonicDrive® Gears

A simple three-element construction combined with the unique operating principle puts extremely high reduction ratio capabilities into a very compact and lightweight package. The high-performance attributes of this gearing technology including, zero-backlash, high-torque-to-weight ratio, compact size, and excellent positional accuracy, are a direct result of the unique operating principles.

The Flexspline is slightly smaller in diameter than the Circular Spline and usually has two fewer teeth than the Circular Spline. The elliptical shape of the Wave Generator causes the teeth of the Flexspline to engage the Circular Spline at two opposite regions across the major axis of the ellipse.

As the Wave Generator rotates the teeth of the Flexspline engage with the Circular Spline at the major axis.

For every 180 degree clockwise movement of the Wave Generator, the Flexspline rotates counterclockwise by one tooth in relation to the Circular Spline.

Each complete clockwise rotation of the Wave Generator results in the Flexspline moving counterclockwise by two teeth from its original position, relative to the Circular Spline. Normally, this motion is taken out as output.

■ Development of HarmonicDrive® Speed Reducers

Harmonic Drive® gears have been evolving since the strain wave gear was first patented in 1955. Our innovative development and engineering teams have led us to significant advances in our gear technology. In 1988, Harmonic Drive successfully designed and manufactured a new tooth profile, the “S” tooth. Since implementing the “S” tooth profile, improvement in life, strength and torsional stiffness have been realized. In the 1990s, we focused engineering efforts on designing gears featuring space savings, higher speed, higher load capacity and higher reliability. Then in the 2000s, significant reduction in size and thickness were achieved, all while maintaining high precision specifications.
FD Series
Differential Gear FD

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Features

Differential gear FD series
The FD series is an extremely compact differential unit that allows you to fine-tune the phase and timing during operation. Like the pancake component sets, the FD series consist of four parts. FD is available as a gear unit with housing or as a component set without a housing. The housing enables additional gears or pulleys to be directly mounted onto it.

Features of FD series
- Pancake component set
- Ultra compact differential unit
- Backlash is very small and unit requires no assembly adjustment
- Very large reduction ratios between the adjusting shaft and the output, it allows precision position adjustment and requires little torque for adjusting the shaft
- Easily installed into OEM equipment
- Component set consists of only four parts and is mounted coaxially

Structure of the FD series

![Diagram of FD series structure](image)

- Unit type (FD-0)
- Component type (FD-2)

<table>
<thead>
<tr>
<th>Part No.</th>
<th>Part name</th>
</tr>
</thead>
<tbody>
<tr>
<td>①</td>
<td>Wave generator</td>
</tr>
<tr>
<td>②</td>
<td>Wave generator plug</td>
</tr>
<tr>
<td>③</td>
<td>Wave generator bearing</td>
</tr>
<tr>
<td>④</td>
<td>Retainer presser</td>
</tr>
<tr>
<td>⑤</td>
<td>Flexspline</td>
</tr>
<tr>
<td>⑥</td>
<td>Circular spline S</td>
</tr>
<tr>
<td>⑦</td>
<td>Circular spline D</td>
</tr>
<tr>
<td>⑧</td>
<td>Ball bearing</td>
</tr>
<tr>
<td>⑨</td>
<td>Internal C-type stop ring</td>
</tr>
<tr>
<td>⑩</td>
<td>Housing</td>
</tr>
<tr>
<td>⑪</td>
<td>Bolt with hexagonal hole</td>
</tr>
</tbody>
</table>

(Note) How to tell circular spline D from circular spline S
The peripheral chamfering of circular spline D is larger than that of circular spline S.
Rotational direction and reduction ratio

The rotational direction is the same as the FB series (Page 105). This section describes how to use the unit as a differential unit. (R indicates the reduction ratio value in the ratings table.)

1. Input: Circular spline S  
   Output: Circular spline D  
   Fixed: Wave generator  
   Reduction ratio (i) is:  
   \[ i = \frac{R+1}{R} \]
   Hence, input rotation: \( N_S \)  
   output speed: \( N_D \)
   \[ N_D = \frac{R+1}{R} N_S \]

When (1) and (2) are combined, the rotational speed of circular spline S is represented as shown on the right.

2. Input: Wave generator  
   Output: Circular spline D  
   Fixed: Circular spline S  
   Reduction ratio (i) is:  
   \[ i = \frac{1}{R} \]
   Hence, input rotation: \( N_w \)  
   output speed: \( N_D \)
   \[ N_D = \frac{1}{R} N_w \]

When (1) and (2) are combined, the rotational speed of circular spline D is represented as shown on the right.

(Note) The sign (+) indicates that the wave generator is turned in the opposite direction to the circular spline S.

(Note) The sign (-) indicates that the wave generator is turned in the opposite direction to the circular spline D.

Fig. 289-1

Fig. 289-2
## Ordering Code

**FD - 20-80-0-G**

<table>
<thead>
<tr>
<th>Series</th>
<th>Size</th>
<th>Ratio *1</th>
<th>Model</th>
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</tr>
<tr>
<td>25</td>
<td>90</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>32</td>
<td>78</td>
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<tr>
<td>40</td>
<td>80</td>
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<tr>
<td>50</td>
<td>80</td>
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<tr>
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<tr>
<td>80</td>
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</tr>
<tr>
<td>100</td>
<td>80</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

*1 The reduction ratio value is based on the following configuration:
Input: wave generator, fixed: circular spline S, output: circular spline D

---

**How To Use**

- **Main motor**: Continuous operation adjustment
- **Adjustment motor**: Variable-speed pulley
- **Drive shaft**: Wave generator

**Usage example**

- **Adjustment**
  - **Fig. 291-1**: Roll (2) Variable-speed pulley
  - **Fig. 291-3**: Z1 Z2 Z3

**Adjustment**

- **Roll (1)**: Z4 CD CS (Z2)
- **Roll (2)**: Z1

**Phase adjustment**

- **Variable-speed pulley**: 
  - **Main motor**: N0. Assuming that the rotation of the variable-speed pulley is Z1 (Calculation formula)
  - **Drive shaft**: Z4 CD CS (Z2) Z1 Z2 Z3

**Adjustment**

- **Main motor**: Z2(CS) Z1
- **Adjusting motor**: Z4(CO)
- **Roll (1)**: Z4 CD CS (Z2) Z1 Z2 Z3
- **Roll (2)**: Z4 CD CS (Z2) Z1 Z2 Z3

**Adjustment**

- **Main motor**: N0. Assuming that the adjusting motor rotates in NW, Z2(CS) Z1
- **Adjusting motor**: N0.

**Brake a unit to adjust the phase of two rolls, normally an**

---

*(Calculation formula)*

- **Adjustment**
  - **Fig. 291-1**: Roll (2) Variable-speed pulley
  - **Fig. 291-3**: Z1 Z2 Z3

**Adjustment**

- **Main motor**: N0. Assuming that the rotation of the variable-speed pulley is Z1 (Calculation formula)
  - **Drive shaft**: Z4 CD CS (Z2) Z1 Z2 Z3
## How To Use

### Usage example

#### Phase adjustment

Brake a unit to adjust the phase of two rolls, normally an adjusting motor, and rotate it in the system: roll (1) → $C_0 \rightarrow C_c \rightarrow$ Roll (2). When the phase of Roll (2) against Roll (1) needs to be adjusted, the adjusting motor should be rotated. Stop the motor after adjustment and return Roll (2) to the original rotation.

(Calculation formula)
When the adjusting motor is fixed, the rotational speed of Roll (2) should be $N_0$. Assuming that the adjusting motor rotates in $N_w$, rotational speed $N$ of the roll is expressed as follows.

$$N = N_0 \pm \frac{1}{R} \left( \frac{Z_2}{Z_1} \right) N_w$$

Formula 291-1

The sign is minus (−) when the wave generator rotates in the same direction as the circular spline. It is plus (+) when the wave generator rotates in the opposite direction.

#### Fine adjustment

This is the method to fine-tune the speed and timing of the drive shaft by the adjusting motor without changing the rotational speed of the main motor.

(Calculation formula)
When the adjusting motor is fixed, the rotational speed of the drive shaft is expressed as follows.

$$N = N_0 \pm \frac{1}{R+1} \left( \frac{Z_2}{Z_1} \right) N_w$$

Formula 291-2

The sign is plus (+) when the wave generator rotates in the same direction as the circular spline. It is minus (−) when the wave generator rotates in the opposite direction.

#### Continuous operation adjustment

This is a unit to continuously make a slight change to the rotational speed of the roll. The rotation of the main motor has the following two routes.

1. $Z_1 \rightarrow Z_2(C_C) \rightarrow Z_2(C_S) \rightarrow Z_1 \rightarrow$ Roll
2. Variable-speed pulley → wave generator → $C_0 \rightarrow Z_2 \rightarrow Z_1 \rightarrow$ Roll

The speed change of the roll is given by (2).

(Calculation formula)
Assuming that the rotation of the variable-speed pulley is zero, the rotational speed of the roll rotated by the main motor is $N_0$. When the rotation of the wave generator, namely the variable-speed pulley, changes from $N_1$ to $N_2$, rotational speed $N$ of the roll is expressed as follows.

$$N = N_0 \pm \frac{1}{R+1} \left( \frac{Z_2}{Z_1} \right) (N_1 \to N_2)$$

Formula 291-3

The sign is plus (+) when the wave generator rotates in the same direction as the circular spline. It is minus (−) when the wave generator rotates in the opposite direction.
Example of assembly

**Paper-cutting machine**

The right-hand figure shows an example of a general application that is used for the mechanism shown below.

**Outline of operation**

Rollers (1), (2) and (3) are interlocked based on the rotation of the cutter. Roller (2) feeds paper for further printing on the printed paper that is then extracted by Roller (1). Roller (2) adjusts the misaligned printing.

Roller (1) adjusts the printed paper so that it will be cut in the correct position by Roller (2). Roller (3) makes further adjustment following Roller (1).

You can change the phase between the rollers by building a Harmonic differential gear in Units (1), (2) and (3) without stopping the unit.

**Printer (film material)**

The following process is essential for printing on elastic material.

1. A device to adjust printing misalignment by expansion and contraction
2. A device to continue tensioning film to prevent wrinkling

**Outline of operation**

The film material is withdrawn by (1). (1) tensions film between (2) and (3) to prevent wrinkling.

(2) tensions film between (2) and (3) to prevent slackening in the printing process (3). In the printing process (4), all rollers from (2) to (5) are used for 6-color printing. Adjustment is made to (6) based on (5), to (7) based on (6) and so on up to (10) by the Harmonic differential gear.

The harmonic differential is built in for all rollers from (1) to (10).

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**Difference between the differential gear and the Harmonic differential gear**

- **Differential gear**
  - As many gears are required in the differential device, the device increases in size, causing problems in design, and is difficult to build in.
  - A unit using a planet gear causes a lot of backlash and is disadvantageous in position and timing precision.
  - It is not easy to fine tune compared to the Harmonic differential gear.
  - Noisy gear sound

- **Harmonic differential gear**
  - As the Harmonic differential gear includes the differential mechanism, it can be designed to be of compact size and is easily built in.
  - As it causes very small backlash, it is advantageous in position and timing precision.
  - As it has a large reduction ratio, it can produce very fine tuning.
  - Very quiet.

The right-hand figure shows an example of a general application that is used for the mechanism shown below.
Difference between the differential gear and the Harmonic differential gear

<table>
<thead>
<tr>
<th></th>
<th>Differential gear</th>
<th>Harmonic differential gear</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>As many gears are required in the differential device, the device increases in</td>
<td>As the Harmonic differential gear includes the differential mechanism, it can be designed</td>
</tr>
<tr>
<td></td>
<td>size, causing problems in design, and is difficult build it in.</td>
<td>to be of compact size and is easily built in.</td>
</tr>
<tr>
<td></td>
<td>A unit using a planet gear causes a lot of backlash and is disadvantageous in</td>
<td>As it causes very small backlash, it is advantageous in position and timing precision.</td>
</tr>
<tr>
<td></td>
<td>position and timing precision.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>It is not easy to fine tune compared to the Harmonic differential gear.</td>
<td>As it has a large reduction ratio, it can produce very fine tuning.</td>
</tr>
<tr>
<td></td>
<td>Noisy gear sound</td>
<td>Very quiet.</td>
</tr>
</tbody>
</table>

That shown in the right-hand figure is a differential gear used in a printer maker. It is an example of very smart, compact design using the Harmonic differential gear.

Conventional differential gear

![Conventional differential gear](Fig. 293-1)

Using the Harmonic differential gear

![Using the Harmonic differential gear](Fig. 293-2)
Example of design

- Multicolor printer Phase adjuster

The figure is an example of a Harmonic differential gear unit (FD-0) built in as a phase adjuster for the roll of a multicolor printer.

The adjusting motor is fixed during normal operation, and the rotation at Z1 is transmitted to Z2 almost at a ratio of 1:1. To adjust the phase of only Roll (2), rotate the adjusting motor to generate a small rotational difference. After adjustment, stop the motor to bring Roll (2) back to the original rotational speed.

Gear selection data

The selection data of the number of teeth, Z1, Z2, Z3, and Z4 of the gear is shown when rotational speed N1 is equal to N2.

\[ N_1 = i \times N_2 \]

where, Zs: the number of teeth of circular spline S
ZD: the number of teeth of circular spline D

Here, \( i = \frac{Z_2}{Z_3} \times \frac{Z_4}{Z_1} \) (or \( R = \frac{Z_1}{Z_2} \)) makes

\[ i = \frac{Z_2}{Z_3} \times \frac{Z_4}{Z_1} \]

Tab. 294-1

<table>
<thead>
<tr>
<th>( i )</th>
<th>( Z_1, Z_2 )</th>
<th>( Z_3, Z_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>18, 18</td>
<td>18, 27</td>
</tr>
<tr>
<td>81</td>
<td>15, 25</td>
<td>21, 26</td>
</tr>
<tr>
<td>120</td>
<td>22, 20, 22</td>
<td>23, 24</td>
</tr>
<tr>
<td>121</td>
<td>22, 24</td>
<td>25, 20</td>
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<td>128</td>
<td>15, 43</td>
<td>16, 32</td>
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<tr>
<td>129</td>
<td>16, 40</td>
<td>33, 43</td>
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<tr>
<td>180</td>
<td>14, 23</td>
<td>21, 24</td>
</tr>
<tr>
<td>161</td>
<td>16, 20</td>
<td>23, 40</td>
</tr>
</tbody>
</table>

(Note) 1. The number of teeth given above is applicable when \( Z_2 \) and \( Z_3 \) are arranged as shown in the figure.
2. The difference in the number of teeth is adjusted to \( Z_1 = Z_2 \leq 3 \) and \( Z_3 = Z_4 \leq 3 \).
3. It is useful to break down to prime numbers to use a different number of teeth.
4. It is not possible to break down to prime numbers for \( R = 79, 96, 103, 131, 208 \) and 258.
Calculation example

This is to calculate the torque required for the number of teeth of the gear, rotational speed, adjustment quantity and adjustment based on the example shown in the right-hand figure (Fig. 267-1).

[Usage condition]
In Figure 267-1
Speed around the roller \( V = 60 \text{m/min} \)
Length around the roller \( L_w = 500 \text{mm} \)
Roller torque \( T_w = 7 \text{kg-m} \)
Rotational speed of the drive shaft \( N_2 = 500 \text{rpm} \)
Rotational speed of the roller \( N_1 = \frac{V}{L_w} = 0.6 = 120 \text{rpm} \)

Under these conditions, select model number 25 of differential gear with reduction ratio \( R=80 \), review whether or not this model number is appropriate, as well as the number of teeth and adjustment torque.

■ The number of teeth of each gear (selection of \( Z_1, Z_2, Z_3 \) and \( Z_4 \))
The total reduction ratio is as follows.
From \( i = \frac{N_1}{N_2} = \frac{Z_2 \cdot Z_3}{Z_1 \cdot Z_4} \cdot \frac{C_b}{C_o} \cdot \frac{Z_1}{Z_2} \)
\[ \frac{Z_2 \cdot Z_3}{Z_1 \cdot Z_4} = \frac{N_1 \cdot C_b}{N_2 \cdot C_o} \]
is calculated.
Here, \( \frac{N_1}{N_2} = \frac{120}{500} = \frac{2 \times 3 \times 5}{2 \times 5^3} \)
\( \frac{C_b}{C_o} = \frac{80}{81} = \frac{2 \times 5}{3^4} \)
\( C_o = \frac{Z_1}{Z_2} \times \frac{Z_3}{Z_4} = \frac{2 \times 3 \times 5}{2 \times 5^3} \times \frac{2 \times 5}{3^4} = \frac{2^3}{3^5} \times \frac{2^7}{3^4} = \frac{2^{10}}{3^9} = \frac{8 \times 4}{15} \times \frac{9}{16} = 16 \times 30 = 36 \)

Hence, \( Z_1 = 30, Z_2 = 16, Z_3 = 36, Z_4 = 16 \)

■ Calculation of rotational speed
The rotational speed of each gear is shown below.
\( Z_1: N_1 = 500 \text{rpm} \)
\( Z_2: N_2 = \frac{Z_1}{Z_2} \times N_1 = 16 \times 500 = 222.2 \text{rpm} \)
\( Z_3: N_3 = \frac{C_b}{C_o} \times N_1 = 81 \times 222.2 = 225 \text{rpm} \)
\( Z_4: N_4 = 120 \text{rpm} \)

■ Adjustment quantity
The misalignment (adjustment quantity), \( \Delta \theta \), at the roller is expressed as follows when the adjusting wave generator rotates once (360°).
\[ \Delta \theta = \frac{Z_2}{Z_1} \cdot \frac{1}{R} \cdot \theta = \frac{16}{30} \times \frac{1}{80} \times 360^\circ = 2.4^\circ \]
Therefore, the adjustment quantity is expressed as follows in the circle.
\[ \Delta \theta = \frac{2.4^\circ}{360^\circ} \times 500 \text{mm} = 3.3 \text{mm} \]

■ Adjustment torque required
The torque required for adjustment is expressed as follows.
\[ T = T_w \cdot \frac{Z_2}{Z_1} \cdot \frac{1}{R} \cdot \frac{1}{\eta} = 7 \text{kg-m} \times \frac{16}{30} \times \frac{1}{80} \times \frac{1}{0.6} = 0.07 \text{kg-m} \]
(\( \eta \): efficiency)
The rated torque at each rotational speed is shown below.

<table>
<thead>
<tr>
<th>Rotational speed rpm</th>
<th>3500</th>
<th>2850</th>
<th>1750</th>
<th>1450</th>
<th>1150</th>
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</tr>
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<td>80</td>
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</tr>
<tr>
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<td>1.1</td>
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<td>137</td>
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<td>157</td>
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<td>1.2</td>
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</tr>
</tbody>
</table>

(Note) 1. Rotational speed: This indicates the rotational speed of the wave generator if used as a reducer. This indicates the relative rotational speed of the wave generator and the circular spline if used as a differential unit.

2. The torque against a rotational speed of 2,500 rpm or less is equal to the torque for 500 rpm.

3. The allowable momentary torque allows up to 200% of the torque at a rotational speed of 1,450 rpm.
## Dimensions (FD-0)

<table>
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<tr>
<th>Symbol</th>
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<tr>
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<td>22.5</td>
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</tr>
<tr>
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<td>M4 × 7</td>
<td>M5 × 8</td>
<td>M6 × 9</td>
<td>M8 × 11</td>
<td>M10 × 13.5</td>
<td>M12 × 23</td>
<td>M12 × 23</td>
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<td>#6008</td>
<td>#6010</td>
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<td>#6018</td>
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Unit: mm

---

**Outline dimensions (FD-0)**

Fig. 297-1
Differential Gear FD

Outline Dimensions (FD-2)

Fig. 298-1

Dimensions (FD-2)

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<td>70</td>
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<td>J</td>
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<th>40</th>
<th>50</th>
<th>65</th>
<th>80</th>
<th>100</th>
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<td>80</td>
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<td>14</td>
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<td>33.3</td>
<td>38.3</td>
<td>43.3</td>
<td>53.8</td>
<td>69</td>
<td>85.4</td>
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<td>S</td>
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<td>53</td>
<td>69</td>
<td>84</td>
<td>105</td>
<td>138</td>
<td>169</td>
<td>211</td>
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<tr>
<td>Mass (kg)</td>
<td></td>
<td>0.6</td>
<td>1.0</td>
<td>2.0</td>
<td>3.6</td>
<td>7.2</td>
<td>14</td>
<td>26</td>
<td>48</td>
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</table>

Unit: mm
### Efficiency

The efficiency of the differential gear unit (FD-0) varies depending on the power transmission route.

1. The efficiency when the power enters from circular spline S (or D) to transmit the rotation to circular spline D (or S)
   - For oil lubrication: About 90%
   - For grease lubrication: About 80%

2. The efficiency for obtaining the input torque required by the wave generator for phase adjustment and to use it as a reducer is shown in Graph 299-1.

### Moment of inertia

The value of GD² of each part is shown in Table 299-2.

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<th>50</th>
<th>65</th>
<th>80</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>I Wave generator (except the outer race of the wave generator)</td>
<td>1.44</td>
<td>3.63</td>
<td>12.9</td>
<td>37.0</td>
<td>112</td>
<td>366</td>
<td>1020</td>
<td>3050</td>
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<tr>
<td>II Circular spline S, D Outer ring of the wave generator</td>
<td>13.7</td>
<td>33.8</td>
<td>125</td>
<td>326</td>
<td>1020</td>
<td>3440</td>
<td>9270</td>
<td>27000</td>
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<tr>
<td>III 1 + II</td>
<td>15.2</td>
<td>37.5</td>
<td>138</td>
<td>363</td>
<td>1140</td>
<td>3810</td>
<td>10300</td>
<td>30100</td>
</tr>
<tr>
<td>IV Support bearing (4)</td>
<td>2.91</td>
<td>8.98</td>
<td>23.4</td>
<td>451</td>
<td>104</td>
<td>205</td>
<td>646</td>
<td>1590</td>
</tr>
<tr>
<td>V Casing (right and left casing total)</td>
<td>52.6</td>
<td>69.0</td>
<td>204</td>
<td>484</td>
<td>1660</td>
<td>6220</td>
<td>15700</td>
<td>43200</td>
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</table>

### Max. allowable rotational speed

The maximum allowable rotational speed means:

1. The rotational speed of the wave generator when used as a reducer
2. The relative rotational speed of the wave generator and the circular spline when used as a differential unit

#### (1) For oil lubrication

<table>
<thead>
<tr>
<th>Size</th>
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<th>25</th>
<th>32</th>
<th>40</th>
<th>50</th>
<th>65</th>
<th>80</th>
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<td>5000</td>
<td>4500</td>
<td>4000</td>
<td>3500</td>
<td>3000</td>
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<td>2000</td>
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#### (2) For grease lubrication

<table>
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<th>80</th>
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<td>3600</td>
<td>3600</td>
<td>3300</td>
<td>3000</td>
<td>2200</td>
<td>2000</td>
<td>1700</td>
</tr>
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</table>
Lost motion and spring constant

See Page 120 for a definition of lost motion and the spring constant. Hysteresis loss and the spring constant of the differential gear is the value when either the wave generator or the circular spline is fixed and a torque is applied to the other circular spline.

<table>
<thead>
<tr>
<th>Size</th>
<th>Lost motion (arc min)</th>
<th>Spring constant (kg/arc min)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>a load (kgm)</td>
<td>Standard product (max.)</td>
</tr>
<tr>
<td>20</td>
<td>0.12</td>
<td>40</td>
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<tr>
<td>25</td>
<td>0.23</td>
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<td>1.73</td>
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<td>3.9</td>
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</tr>
<tr>
<td>80</td>
<td>7.4</td>
<td>26</td>
</tr>
<tr>
<td>100</td>
<td>14.4</td>
<td>24</td>
</tr>
</tbody>
</table>
Design Guide

Precaution on handling

The casing and the roller bearing in using a component type (FD-2) as a differential unit should be pursuant to the unit type (FD-0).

Precautions on assembly

The FD gear may generate vibration and abnormal sound due to problems during assembly. Perform assembly based on the FB series precautions (Page 109, Fig. 109-2).

Lubrication

There are two types of lubrication; oil lubrication and grease lubrication. Although oil lubrication is common, grease lubrication is applicable to intermittent operation.

Oil lubrication

1. Type of lubricant

See Page 018 for lubrication details.

2. Oil quantity

The oil level shall be the position shown in Table 301-1.

<table>
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<tr>
<th>Size</th>
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<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>12</td>
<td>15</td>
<td>31</td>
<td>38</td>
<td>44</td>
<td>62</td>
<td>75</td>
<td>94</td>
</tr>
</tbody>
</table>

Grease lubrication

Different from oil lubrication, as a cooling effect is not expected from grease lubrication, it is only available for short operation.

- Operating condition: ED% ≤ 10% or less, continuous operation for 10 minutes or less, the maximum allowable input rotational speed in Table 271-4 or less
- Recommended grease: Harmonic Grease SK-1A

Note: If you use the product over ED% or the maximum permissible rotational speed, the grease will deteriorate, will not work as a lubricating mechanism and will result in damaging the reducer earlier. Extreme care should be taken.

Please consider the unit type since unit type (FD-0) also comes in grease sealed type (NIPPON KOYU LTD. MP No. 2).
### Engineering Data

#### Tooth profile
- S tooth profile

#### Rotational direction and reduction ratio
- Cup style
- Silk hat style
- Pancake style

#### Rating table definitions

#### Life

#### Torque limits

#### Product sizing and selection

#### Lubrication
- Grease lubricant
- Oil lubricant
- Lubricant for special environments

#### Torsional stiffness

#### Positional accuracy

#### Vibration

#### Starting torque

#### Backdriving torque

#### No-load running torque

#### Efficiency

#### Design guidelines
- Design guideline
- Bearing support of the input and output shafts
- Wave Generator

#### Assembly guidelines
- Sealing
- Assembly Precautions
- "dedoidal" state

#### Checking output bearing
- Checking procedure
- How to calculate the maximum moment load
- How to calculate the average load
- How to calculate the radial load coefficient (x) and axial load coefficient (Y)
- How to calculate life
- How to calculate the life under oscillating movement
- How to calculate the static safety coefficient

---

**Fig. 009-1** Engaged area of teeth
**Fig. 009-2** Engaged route of teeth

**Tooth Profile**

Harmonic Drive developed a unique gear tooth profile that optimizes the tooth engagement. It has a special curved surface unique to the S tooth profile that allows continuous contact with the tooth profile. It also alleviates the concentration of stress by widening the width of the tooth groove against the tooth thickness and enlarging the radius on the bottom. This tooth profile (the "S tooth") enables up to 30% of the total number of teeth to be engaged simultaneously. Additionally, the large tooth root radius increases the tooth strength compared with an involute tooth. This technological innovation results in high torque, high torsional stiffness, long life, and smooth rotation.
Tooth Profile

- **S tooth profile**

Harmonic Drive developed a unique gear tooth profile that optimizes the tooth engagement. It has a special curved surface unique to the S tooth profile that allows continuous contact with the tooth profile. It also alleviates the concentration of stress by widening the width of the tooth groove against the tooth thickness and enlarging the radius on the bottom. This tooth profile (the “S tooth”) enables up to 30% of the total number of teeth to be engaged simultaneously.

Additionally the large tooth root radius increases the tooth strength compared with an involute tooth. This technological innovation results in high torque, high torsional stiffness, long life and smooth rotation.

*Patented*
**Rotational direction and reduction ratio**

### Cup Style

Series: CSG, CSF, CSD, CSF-mini

#### Rotational direction

1. **Reducer**
   - Input: Circular Spline
   - Output: Flexspline
   - Fixed: Wave Generator
   - Reduction ratio: \( i = \frac{R_1}{R_2} \)

2. **Reducer**
   - Input: Circular Spline
   - Output: Flexspline
   - Fixed: Wave Generator
   - Reduction ratio: \( i = \frac{R_1}{R_2} \)

3. **Reducer**
   - Input: Flexspline
   - Output: Circular Spline
   - Fixed: Wave Generator
   - Reduction ratio: \( i = \frac{R_1}{R_2} \)

4. **Overdrive**
   - Input: Circular Spline
   - Output: Flexspline
   - Fixed: Wave Generator
   - Reduction ratio: \( i = \frac{R_1}{R_2} \)

5. **Overdrive**
   - Input: Circular Spline
   - Output: Flexspline
   - Fixed: Wave Generator
   - Reduction ratio: \( i = \frac{R_1}{R_2} \)

6. **Overdrive**
   - Input: Circular Spline
   - Output: Flexspline
   - Fixed: Wave Generator
   - Reduction ratio: \( i = \frac{R_1}{R_2} \)

7. **Differential**
   - When all of the wave generator, the flexspline and the circular spline rotate, combinations (1) through (6) are available.

### Silk hat

Series: SHG, SHF, SHD

#### Rotational direction

1. **Reducer**
   - Input: Circular Spline
   - Output: Flexspline
   - Fixed: Wave Generator
   - Reduction ratio: \( i = \frac{R_1}{R_2} \)

2. **Reducer**
   - Input: Circular Spline
   - Output: Flexspline
   - Fixed: Wave Generator
   - Reduction ratio: \( i = \frac{R_1}{R_2} \)

3. **Reducer**
   - Input: Flexspline
   - Output: Circular Spline
   - Fixed: Wave Generator
   - Reduction ratio: \( i = \frac{R_1}{R_2} \)

4. **Overdrive**
   - Input: Circular Spline
   - Output: Flexspline
   - Fixed: Wave Generator
   - Reduction ratio: \( i = \frac{R_1}{R_2} \)

5. **Overdrive**
   - Input: Circular Spline
   - Output: Flexspline
   - Fixed: Wave Generator
   - Reduction ratio: \( i = \frac{R_1}{R_2} \)

6. **Overdrive**
   - Input: Circular Spline
   - Output: Flexspline
   - Fixed: Wave Generator
   - Reduction ratio: \( i = \frac{R_1}{R_2} \)

7. **Differential**
   - When all of the wave generator, the flexspline and the circular spline rotate, combinations (1) through (6) are available.
### Rotational direction

1. **Reducer**
   - Input: Wave Generator
   - Output: Circular Spline D
   - Reduced ratio: $i = \frac{R_1}{R_2}$
   - Example: Number of teeth of the Flexspline: 200
   - Number of teeth of the Circular Spline: 202
   - Reduction ratio: $i = \frac{200}{200} = 1$

2. **Reducer**
   - Input: Wave Generator
   - Output: Circular Spline S
   - Reduced ratio: $i = \frac{R_1}{R_2}$
   - Example: Number of teeth of the Flexspline: 200
   - Number of teeth of the Circular Spline: 202
   - Reduction ratio: $i = \frac{200-202}{200} = -1$

3. **Reducer**
   - Input: Circular Spline D
   - Output: Wave Generator
   - Reduced ratio: $i = \frac{R_1}{R_2}$
   - Example: Number of teeth of the Flexspline: 200
   - Number of teeth of the Circular Spline: 202
   - Reduction ratio: $i = \frac{202-202}{202} = 1$

4. **Overdrive**
   - Input: Circular Spline D
   - Output: Wave Generator
   - Reduced ratio: $i = \frac{R_1}{R_2}$
   - Example: Number of teeth of the Flexspline: 200
   - Number of teeth of the Circular Spline: 202
   - Reduction ratio: $i = \frac{202-202}{202} = 1$

5. **Overdrive**
   - Input: Circular Spline S
   - Output: Wave Generator
   - Reduced ratio: $i = \frac{R_1}{R_2}$
   - Example: Number of teeth of the Flexspline: 200
   - Number of teeth of the Circular Spline: 202
   - Reduction ratio: $i = \frac{202-202}{202} = 1$

6. **Overdrive**
   - Input: Circular Spline D
   - Output: Wave Generator
   - Reduced ratio: $i = \frac{R_1}{R_2}$
   - Example: Number of teeth of the Flexspline: 200
   - Number of teeth of the Circular Spline: 202
   - Reduction ratio: $i = \frac{202-202}{202} = 1$

7. **Differential**
   - When all of the Wave Generator, the Circular Spline S and the Circular Spline D rotate, Combinations (1) through (6) are available.

---

### Reduction ratio

The reduction ratio is determined by the number of teeth of the Flexspline and the Circular Spline.

**Input**: Wave Generator  
**Output**: Circular Spline

- **Fixed**: Wave Generator
- **Output**: Circular Spline

- **Input**: Wave Generator  
  - **Output**: Circular Spline
  - Reduced ratio: $i = \frac{1}{R_1} = \frac{Zf-Zc}{Zf}$

- **Input**: Wave Generator  
  - **Output**: Flexspline
  - Reduced ratio: $i = \frac{1}{R_1} = \frac{Zf-Zc}{Zc}$

- **Input**: Wave Generator  
  - **Output**: Flexspline
  - Reduced ratio: $i = \frac{1}{R_1} = \frac{Zf-Zc}{Zc}$

$R_1$ indicates the reduction ratio value from the ratings table.
Rating Table Definitions

See the corresponding pages of each series for values.

■ Rated torque
Rated torque indicates allowable continuous load torque at rated input speed.

■ Limit for Repeated Peak Torque
(see Graph 12-1)
During acceleration and deceleration the Harmonic Drive® gear experiences a peak torque as a result of the moment of inertia of the output load. The table indicates the limit for repeated peak torque.

■ Limit for Average Torque
In cases where load torque and input speed vary, it is necessary to calculate an average value of load torque. The table indicates the limit for average torque. The average torque calculated must not exceed this limit. (calculation formula: Page 14)

■ Limit for Momentary Peak Torque
(see Graph 12-1)
The gear may be subjected to momentary peak torques in the event of a collision or emergency stop. The magnitude and frequency of occurrence of such peak torques must be kept to a minimum and they should, under no circumstance, occur during normal operating cycle. The allowable number of occurrences of the momentary peak torque may be calculated by using formula 13-1.

■ Maximum Average Input Speed
Maximum Input Speed
Do not exceed the allowable rating. (calculation formula of the average input speed: Page 14).

■ Moment of Inertia
The rating indicates the moment of inertia reflected to the gear input.

Life

■ Life of the wave generator
The life of a gear is determined by the life of the wave generator bearing. The life may be calculated by using the input speed and the output load torque.

<table>
<thead>
<tr>
<th>Life</th>
</tr>
</thead>
<tbody>
<tr>
<td>Series name</td>
</tr>
<tr>
<td>CSF, CSD, SHF, SHD, CSF-mini</td>
</tr>
<tr>
<td>CSG, SHG</td>
</tr>
<tr>
<td>L10 (average life)</td>
</tr>
<tr>
<td>7,000 hours</td>
</tr>
<tr>
<td>10,000 hours</td>
</tr>
<tr>
<td>L10 (average life)</td>
</tr>
<tr>
<td>35,000 hours</td>
</tr>
<tr>
<td>50,000 hours</td>
</tr>
</tbody>
</table>

* Life is based on the input speed and output load torque from the rating table.

Calculation formula for Rated Lifetime

\[ L_h = L_10 \left( \frac{T_r}{T_{av}} \right) \cdot \left( \frac{N_r}{N_{av}} \right) \]

<table>
<thead>
<tr>
<th>Table 012-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ln</td>
</tr>
<tr>
<td>Life of L10 or L10</td>
</tr>
<tr>
<td>Tr</td>
</tr>
<tr>
<td>Rated torque</td>
</tr>
<tr>
<td>Nr</td>
</tr>
<tr>
<td>Rated input speed</td>
</tr>
<tr>
<td>Tav</td>
</tr>
<tr>
<td>Average load torque on the output side (calculation formula: Page 14)</td>
</tr>
<tr>
<td>Nav</td>
</tr>
<tr>
<td>Average input speed (calculation formula: Page 14)</td>
</tr>
</tbody>
</table>

Relative torque rating

Graph 012-2

* Lubricant life not taken into consideration in the graph described above.
* Use the graph above as reference values.
Torque Limits

**Strength of flexspline**

The Flexspline is subjected to repeated deflections, and its strength determines the torque capacity of the Harmonic Drive® gear. The values given for Rated Torque at Rated Speed and for the allowable Repeated Peak Torque are based on an infinite fatigue life for the Flexspline. The torque that occurs during a collision must be below the momentary peak torque (impact torque). The maximum number of occurrences is given by the equation below.

Formula 013-1

\[ N = \frac{1.0 \times 10^4}{2 \times \frac{n \times t}{60}} \]

Where:
- \( N \) =允許の寄生モーメントの数 (cycles)
- \( n \) = 円筒軸の回転数 (rpm)
- \( t \) = 作用する力が作用する時間 (秒)

The torque that occurs during a collision must be below the momentary peak torque (impact torque). The maximum number of occurrences is given by the equation below.

**Calculation formula**

\[ N = \frac{1.0 \times 10^4}{2 \times \frac{n \times t}{60}} \]

<table>
<thead>
<tr>
<th>Allowable occurrences</th>
<th>N occurrences</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time that impact torque is applied</td>
<td>t sec</td>
</tr>
<tr>
<td>Rotational speed of the wave generator</td>
<td>n rpm</td>
</tr>
</tbody>
</table>

The flexspline bends two times per one revolution of the wave generator.

**Buckling torque**

When a highly excessive torque (16 to 17 times rated torque) is applied to the output with the input stationary, the flexspline may experience plastic deformation. This is defined as buckling torque.

* See the corresponding pages of each series for buckling torque values.

**Ratcheting torque**

When excessive torque (8 to 9 times rated torque) is applied while the gear is in motion, the teeth between the Circular Spline and Flexspline may not engage properly. This phenomenon is called ratcheting and the torque at which this occurs is called ratcheting torque. Ratcheting may cause the Flexspline to become non-concentric with the Circular Spline. Operating in this condition may result in shortened life and a Flexspline fatigue failure.

* See the corresponding pages of each series for ratcheting torque values.

Caution

When ratcheting occurs, the teeth may not be correctly engaged and become out of alignment as shown in Figure 013-1. Operating the drive in this condition will cause vibration and damage the flexspline.

Once ratcheting occurs, the teeth wear excessively and the ratcheting torque may be lowered.

Caution

If the number of occurrences is exceeded, the Flexspline may experience a fatigue failure.

Warning

When the flexspline buckles, early failure of the HarmonicDrive® gear will occur.

"Dedoidal" condition.
**Product Sizing & Selection**

In general, a servo system rarely operates at a continuous load and speed. The input rotational speed, load torque change and comparatively large torque are applied at start and stop. Unexpected impact torque may be applied. These fluctuating load torques should be converted to the average load torque when selecting a model number. As an accurate cross roller bearing is built in the direct external load support (output flange), the maximum moment load, life of the cross roller bearing and the static safety coefficient should also be checked.

### Checking the application motion profile

Review the application motion profile. Check the specifications shown in the figure below.

![Graph 14-1](image)

- **Load torque**
  - $T_1$ (Nm)
- **Time**
  - $t_1$ (sec)
- **Output rotational speed**
  - $n_1$ (rpm)

#### Normal operation pattern

- Starting (acceleration): $T_1, t_1, n_1$
- Steady operation: $T_2, t_2, n_2$
- Stopping (deceleration): $T_3, t_3, n_3$
- Dwell: $T_4, t_4, n_4$

#### Maximum rotational speed

- Max. output speed: $n_1 \text{ max}$
- Max. input rotational speed (Restricted by motors): $n_1 \text{ max}$

#### Emergency stop torque

- When impact torque is applied: $T_s, t_s, n_s$

#### Required life

- $L_{10} = L$ (hours)

---

**Flowchart for selecting a size**

Please use the flowchart shown below for selecting a size. Operating conditions must not exceed the performance ratings.

1. **Calculate the average load torque applied on the output side from the application motion profile: $T_{av}$ (Nm).**
   
   \[ T_{av} = \frac{3}{n_1} \sum_{i=1}^{n} \frac{T_i}{t_i + t_{i+1}} \]

2. **Make a preliminary model selection with the following conditions.**
   
   $T_{max}$, $n_{av}$, $R$

3. **Calculate the average output rotational speed: $n_{av}$ (rpm).**

4. **Obtain the reduction ratio ($R$).** A limit is placed on “$n_{max}$” by motors.

5. **Calculate the average input rotational speed from the average output rotational speed ($n_{av}$) and the reduction ratio ($R$).**

6. **Calculate the maximum input rotational speed from the max. output rotational speed ($n_{max}$) and the reduction ratio ($R$).**

7. **Check whether the preliminary model number satisfies the following condition from the rating table.**
   
   $n_{av} \leq n_{av} \text{ max}$
   
   $n_{max} \leq n_{max} \text{ max}$

8. **Check whether $T_1$ and $T_3$ are less than the repeated peak torque specification.**

9. **Check whether $T_1$ is less than the the momentary peak torque specification.**

10. **Calculate ($N_s$) the allowable number of rotations during impact torque.**
    
    \[ N_s = \frac{10^4}{2 \times n_s \times R} \]

11. **Calculate the lifetime.**
    
    \[ L_{10} = 7000 \left( \frac{Tr}{T_{av}} \right)^{\frac{3}{n_{av}} \text{ (hours)}} \]

12. **Check whether the calculated life is equal to or more than the life of the wave generator (see Page 13).**

The model number is confirmed.
### Example of model number selection

#### Value of each application motion profile

<table>
<thead>
<tr>
<th>Load torque</th>
<th>T1 (Nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time</td>
<td>t1 (sec)</td>
</tr>
<tr>
<td>Output speed</td>
<td>n1 (rpm)</td>
</tr>
</tbody>
</table>

#### Normal operation pattern

| Starting (acceleration) | T1 = 400 Nm, t1 = 0.3sec, n1 = 7rpm |
| Steady operation (constant velocity) | T2 = 320 Nm, t2 = 3sec, n2 = 14rpm |
| Stopping (deceleration) | T3 = 200 Nm, t3 = 0.4sec, n3 = 7rpm |
| Dwell          | T4 = 0 Nm, t4 = 0.2 sec, n4 = 0 rpm |

#### Maximum rotational speed

- Max. output speed: no max = 14 rpm
- Max. input speed: ni max = 1800 rpm

#### Emergency stop torque

When impact torque is applied:

- Ts = 500 Nm, ts = 0.15 sec, ns = 14 rpm

#### Required life

L10 = 7000 (hours)

### Calculation

1. **Calculate the average load torque to the output side based on the application motion profile:**

   \[
   T_{av} = \frac{3 \times 7 \text{ rpm} \cdot 0.3 \text{ sec} \cdot |400 \text{Nm}| + 14 \text{ rpm} \cdot 3 \text{ sec} \cdot |320 \text{Nm}| + 7 \text{ rpm} \cdot 0.4 \text{ sec} \cdot |200 \text{Nm}|}{7 \text{ rpm} \cdot 0.3 \text{ sec} + 14 \text{ rpm} \cdot 3 \text{ sec} + 7 \text{ rpm} \cdot 0.4 \text{ sec}}
   \]

   **Result:** T_{av} = 319 Nm ≤ 620 Nm

   (Limit for average torque for model number CSF-40-120-2A-GR: See the rating table on Page 39.)

   Thus, CSF-40-120-2A-GR is tentatively selected.

2. **Calculate the average output rotational speed:**

   \[
   \text{no av} = \frac{7 \text{ rpm} \cdot 0.3 \text{ sec} + 14 \text{ rpm} \cdot 3 \text{ sec} + 7 \text{ rpm} \cdot 0.4 \text{ sec}}{0.3 \text{ sec} + 3 \text{ sec} + 0.4 \text{ sec} + 0.2 \text{ sec}} = 12 \text{ rpm}
   \]

3. **Calculate the allowable number (Ns) rotation during impact torque and confirm 1.0x10^4:**

   \[
   N_S = \frac{10^4}{2 \times 14 \text{ rpm} \cdot 120 \text{ sec}/60 \times 0.15 \text{ sec}} = 1190 \leq 1.0 \times 10^4
   \]

4. **Calculate the lifetime:**

   \[
   L_{10} = 7000 \left( \frac{294 \text{ Nm}}{319 \text{ Nm}} \right) \left( \frac{2000 \text{ rpm}}{1440 \text{ rpm}} \right) \text{ (hours)}
   \]

   **Result:** L_{10} = 7610 hours ≥ 7000 (life of the wave generator: L_{10})

### Conclusion

The selection of model number CSF-40-120-2-GR is confirmed from the above calculations.
Lubrication


Grease lubricant and oil lubricant are available for lubricating the component sets and SHD gear unit. It is extremely important to properly grease your component sets and SHD gear unit. Proper lubrication is essential for high performance and reliability. Harmonic Drive® component sets are shipped with a rust- preventive oil. The characteristics of the lubricating grease and oil types approved by Harmonic Drive are not changed by mixing with the preservation oil. It is therefore not necessary to remove the preservation oil completely from the gear components. However, the mating surfaces must be degreased before the assembly.

Gear Units: CSG/CSF 2UH and 2UH-LW; CSD-2UF and -2UH; SHG/SHF-2UH and 2UH- LW; SHG/SHF-2UJ; CSF Supermini, CSF Mini, and CSF-2UP.

Grease lubricant is standard for lubricating the gear units. You do not need to apply grease during assembly as the product is lubricated and shipped. See Page 19 for using lubricant beyond the temperature range in table 16-2.

* Contact us if you want consistency zero (NLGI No.0) for maintenance reasons.

### Types of lubricant

**Harmonic Grease® SK-1A**

This grease was developed for Harmonic Drive® gears and features good durability and efficiency.

**Harmonic Grease® SK-2**

This grease was developed for small sized Harmonic Drive® gears and features smooth rotation of the Wave Generator since high pressure additive is liquefied.

**Harmonic Grease® 4B No.2**

This has been developed exclusively for the CSF and CSG and features long life and can be used over a wide range of temperature.

(Note)

1. Grease lubrication must have proper sealing, this is essential for 4B No.2. Rotating part: Oil seal with spring is needed. Mating part: O ring or seal adhesive is needed.

2. The grease has the highest deterioration rate in the region where the grease is subjected to the greatest shear (near wave generator).

Its viscosity is between JIS No.0 and No.00 depending on the operation.

### Grease specification

<table>
<thead>
<tr>
<th>Grease</th>
<th>SK-1A</th>
<th>SK-2</th>
<th>4B No.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base oil</td>
<td>Refine oil</td>
<td>Refine oil</td>
<td>Composite hydrocarbon oil</td>
</tr>
<tr>
<td>Base Viscosity cSt (25°C)</td>
<td>265 to 295</td>
<td>265 to 295</td>
<td>290 to 320</td>
</tr>
<tr>
<td>Thickening agent</td>
<td>Lithium soap base</td>
<td>Lithium soap base</td>
<td>Urea</td>
</tr>
<tr>
<td>NLGI consistency No.</td>
<td>No. 2</td>
<td>No. 2</td>
<td>No. 1.5</td>
</tr>
<tr>
<td>Additive</td>
<td>Extreme-pressure additive, others</td>
<td>Extreme-pressure additive, others</td>
<td>Extreme-pressure additive, others</td>
</tr>
<tr>
<td>Drop Point</td>
<td>197°C</td>
<td>198°C</td>
<td>247°C</td>
</tr>
<tr>
<td>Appearance</td>
<td>Yellow</td>
<td>Green</td>
<td>Light yellow</td>
</tr>
<tr>
<td>Storage life</td>
<td>5 years in sealed condition</td>
<td>5 years in sealed condition</td>
<td>5 years in sealed condition</td>
</tr>
</tbody>
</table>

### Grease characteristics

<table>
<thead>
<tr>
<th>Grease</th>
<th>SK-1A</th>
<th>SK-2</th>
<th>4B No.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Durability</td>
<td>☐</td>
<td>☐</td>
<td>☒</td>
</tr>
<tr>
<td>Fretting resistance</td>
<td>☐</td>
<td>☐</td>
<td>☒</td>
</tr>
<tr>
<td>Low-temperature performance</td>
<td>☐</td>
<td>☐</td>
<td>☒</td>
</tr>
<tr>
<td>Grease leakage</td>
<td>☐</td>
<td>☐</td>
<td>☒</td>
</tr>
</tbody>
</table>

* Oil lubrication is required for component-sets size 50 or larger with a reduction ratio of 50:1.

### Temperature

<table>
<thead>
<tr>
<th>Grease</th>
<th>Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>SK-1A</td>
<td>0°C to + 40°C</td>
</tr>
<tr>
<td>SK-2</td>
<td>0°C to + 40°C</td>
</tr>
<tr>
<td>4B No.2</td>
<td>–10°C to + 70°C</td>
</tr>
</tbody>
</table>

* The hottest section should not be more than 40° above the ambient temperature.

**Note:** The three basic components of the gear - the Flex spline, Wave Generator and Circular Spline - are matched and serialized in the factory. Depending on the product they are either greased or prepared with preservation oil. Then the individual components are assembled. If you receive several units, please be careful not to mix the matched components. This can be avoided by verifying that the serial numbers of the assembled gear components are identical.

### Compatible grease by size

Compatible grease varies depending on the size and reduction ratio. See the following compatibility table. We recommend SK-1A and SK-2 for general use.

#### Ratios 30:1

<table>
<thead>
<tr>
<th>Size</th>
<th>8</th>
<th>11</th>
<th>14</th>
<th>17</th>
<th>20</th>
<th>25</th>
<th>32</th>
</tr>
</thead>
<tbody>
<tr>
<td>SK-1A</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>SK-2</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>4B No.2</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
</tbody>
</table>

#### Ratios 50:1* and above

<table>
<thead>
<tr>
<th>Size</th>
<th>40</th>
<th>45</th>
<th>50</th>
<th>58</th>
<th>65</th>
<th>80</th>
<th>90</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>SK-1A</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>SK-2</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>4B No.2</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
</tbody>
</table>

* Standard grease
  * Semi-standard grease
  * Recommended grease for long life and high load

* Oil lubrication is required for component-sets size 50 or larger with a reduction ratio of 50:1.

---

**Graph 017-1**

**Table 016-5**

**Table 016-6**

**Table 016-3**

**Table 016-4**

**Table 016-1**

**Table 016-2**
When to replace grease

The wear characteristics of the gear are strongly influenced by the condition of the grease lubrication. The condition of the grease is affected by the ambient temperature. The graph 017-1 shows the maximum number of input rotations for various temperatures. This graph applies to applications where the average load torque does not exceed the rated torque.

Note: Recommended Grease: SK-1A or SK-2

When to replace grease: \( L_{tth} \) (when the average load torque is equal to or less than the rated torque)

![Graph 017-1]

Calculation formula when the average load torque exceeds the rated torque

Formula 017-1

\[
L_{at} = L_{sth} \times \left( \frac{Tr}{T_{av}} \right)^3
\]

Other precautions

1. Avoid mixing different kinds of grease. The gear should be in an individual case when installed.

2. Please contact us when you use HarmonicDrive® gears at constant load or in one direction continuously, as it may cause lubrication problems.

3. Grease leakage. A sealed structure is needed to maintain the high durability of the gear and prevent grease leakage.

See the corresponding pages of the design guide of each series for “Recommended minimum housing clearance,” Application guide” and “Application quantity.”
Precautions on using Harmonic Grease® 4B No.2

Harmonic Grease® 4B No.2 lubrication is ideally suited for Harmonic Drive® gears.

(1) Apply the grease to each contacting joint at the beginning of operation.
(2) Remove any contaminants created by abrasion during running-in period.

See the corresponding pages of the design guide of each series for “recommended minimum housing clearance,” Application guide” and “Application quantity.”

Precautions

(1) Stir Grease
When storing Harmonic Grease 4B No.2 lubrication in the container, it is common for the oil to weep from the thickener. Before greasing, stir the grease in the container to mix and soften.

(2) Aging (running-in)
The aging before the main operation softens the applied grease. More effective greasing performance can be realized when the grease is distributed around each contact surface.

Therefore, the following aging methods are recommended.
• Keep the internal temperature at 80°C or cooler. Do not start the aging at high temperature rapidly.
• Input rotational speed should be 1000rpm to 3000rpm. However, the lower rotational speed of 1000rpm is more effective.
• The time required for aging is 20 minutes or longer.
• Operation range for aging: Keep the output rotational angle as large as possible.

Contact us if you have any questions for handling Harmonic Grease 4B No.2 lubrication.

Note: Strict sealing is required to prevent grease leakage.

Oil lubricant

Types of oil
The specified standard lubricant is “Industrial gear oil class-2 (extreme pressure) ISO VG68.”
We recommend the following brands as a commercial lubricant.

<table>
<thead>
<tr>
<th>Standard</th>
<th>Mobil Oil</th>
<th>Exxon</th>
<th>Shell</th>
<th>COSMO Oil</th>
<th>Japan Energy</th>
<th>NIPPON Oil</th>
<th>Idemitsu Kosoan</th>
<th>General Oil</th>
<th>Klüber</th>
</tr>
</thead>
<tbody>
<tr>
<td>Industrial gear oil class-2</td>
<td>Mobilgear 600XP68</td>
<td>Spartan EP68</td>
<td>Omaha Oil 68</td>
<td>Cosmo gear SE68</td>
<td>ES gear G68</td>
<td>Bonock M68, Bonock AX68</td>
<td>Daphne super gear LW68</td>
<td>General Oil SP gear roll 68</td>
<td>Syntheso D-68EP</td>
</tr>
</tbody>
</table>

When to replace oil
First time 100 hours after starting operation
Second time or after Every 1000 operation hours or every 6 months
Note that you should replace the oil earlier than specified if the operating condition is demanding.

See the corresponding pages of the design guide of each series for specific details.

Other precautions
1. Avoid mixing different kinds of oil. The gear should be in an individual case when installed.

2. When you use size 50 or above at max allowable input speed, please contact us as it may cause lubrication problems.

* Oil lubrication is required for component-sets size 50 or larger with a reduction ratio of 50:1.
Lubricant for special environments

When the ambient temperature is special (other than the "temperature range of the operating environment" on Page 016-2), you should select a lubricant appropriate for the operating temperature range.

Harmonic Grease 4B No.2

<table>
<thead>
<tr>
<th>Type of lubricant</th>
<th>Operating temperature range</th>
<th>Available temperature range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grease</td>
<td>-10°C to +110°C</td>
<td>-50°C to +130°C</td>
</tr>
</tbody>
</table>

High temperature lubricant

<table>
<thead>
<tr>
<th>Type of lubricant</th>
<th>Lubricant and manufacturer</th>
<th>Available temperature range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grease</td>
<td>Mobil grease 28: Mobil Oil</td>
<td>-5°C to +160°C</td>
</tr>
<tr>
<td>Oil</td>
<td>Mobil SHC-626: Mobil Oil</td>
<td>-5°C to +140°C</td>
</tr>
</tbody>
</table>

Low temperature lubricant

<table>
<thead>
<tr>
<th>Type of lubricant</th>
<th>Lubricant and manufacturer</th>
<th>Available temperature range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grease</td>
<td>Multemp SH-KII: Kyodo Oil</td>
<td>-30°C to +50°C</td>
</tr>
<tr>
<td></td>
<td>Isoflex LDS-18 special A: KLÜBER</td>
<td>-25°C to +80°C</td>
</tr>
<tr>
<td>Oil</td>
<td>SH-200-100CS: Toray Silicon</td>
<td>-40°C to +140°C</td>
</tr>
<tr>
<td></td>
<td>Syntheso D-32EP: KLÜBER</td>
<td>-25°C to +90°C</td>
</tr>
</tbody>
</table>

Harmonic Grease 4B No.2

The operating temperature range of Harmonic Grease 4B No.2 lubrication is the temperature at the lubricating section with the performance and characteristics of the gear taken into consideration. (It is not ambient temperature.)

As the available temperature range indicates the temperature of the independent lubricant, restriction is added on operating conditions (such as load torque, rotational speed and operating cycle) of the gear. When the ambient temperature is very high or low, materials of the parts of the gear need to be reviewed for suitability. Contact us if operating in high temperature.

Harmonic Grease 4B No.2 can be used in the available temperature range shown in table 019-1. However, input running torque will increase at low temperatures, and grease life will be decreased at high temperatures due to oxidation and lubricant degradation.
Torsional Stiffness

Stiffness and backlash of the drive system greatly affects the performance of the servo system. Please perform a detailed review of these items before designing your equipment and selecting a model number.

- **Stiffness**
  Fixing the input side (wave generator) and applying torque to the output side (flex spline) generates torsion almost proportional to the torque on the output side. Figure 018-1 shows the torsional angle at the output side when the torque applied on the output side starts from zero, increases up to +T0 and decreases down to -T0. This is called the “Torque – torsional angle diagram,” which normally draws a loop of 0 – A – B – A’ – B’ – A. The slope described in the “Torque – torsional angle diagram” is represented as the spring constant for the stiffness of the HarmonicDrive® gear (unit: Nm/rad).

  As shown in Figure 020-1, this “Torque – torsional angle diagram” is divided into 3 regions, and the spring constants in the area are represented by K1, K2 and K3.

  - K1 --- The spring constant when the torque changes from [zero] to [T1]
  - K2 --- The spring constant when the torque changes from [T1] to [T2]
  - K3 --- The spring constant when the torque changes from [T2] to [T3]

- **Example for calculating the torsion angle**
  The torsion angle (θ) is calculated here using CSF-25-100-2A-GR as an example.

  **When the applied torque is T1 or less, the torsion angle θ1 is calculated as follows:**
  When the load torque T1=2.9 Nm
  \[ \theta_1 = \frac{T_1}{K_1} = \frac{2.9/3.1 \times 10^4}{9.4 \times 10^{-1}} = 9.4 \times 10^{-5} \text{ rad} (0.33 \text{ arc min}) \]

  **When the applied torque is between T1 and T2, the torsion angle θ2 is calculated as follows:**
  When the load torque is T2=39 Nm
  \[ \theta_2 = \frac{\theta_1 + (T_2 - T_1)/K_2}{4.4 \times 10^{-2} + (39-14)/5.0 \times 10^4} = 9.4 \times 10^{-1} \text{ rad} (3.2 \text{ arc min}) \]

  When a bidirectional load is applied, the total torsion angle will be 2 x θl plus hysteresis loss.

  * The torsion angle calculation is for the gear component set only and does not include any torsional windup of the input shaft.

  **Note:** See p.120 for torsional stiffness for pancake gearing.

- **Hysteresis loss (Silk hat and cup style only)**
  As shown in Figure 020-1, when the applied torque is increased to the rated torque and is brought back to [zero], the torsional angle does not return exactly back to the zero point. This small difference (B – B’) is called hysteresis loss.

  - See the corresponding page of each series for the hysteresis loss value.

- **Backlash (Silk hat and cup style only)**
  Hysteresis loss is primarily caused by internal friction. It is a very small value and will vary roughly in proportion to the applied load. Because HarmonicDrive® gears have zero backlash, the only true backlash is due to the clearance in the Oldham coupling, a self-aligning mechanism used on the wave generator. Since the Oldham coupling is used on the input, the backlash measured at the output is extremely small (arc-seconds) since it is divided by the gear reduction ratio.
Positional Accuracy

Positional Accuracy values represent the difference between the theoretical angle and the actual angle of output for any given input. The values shown in the table are maximum values. 

See the corresponding pages of each series for transmission accuracy values.

Example of measurement

![Graph of Positional Accuracy](image)

Table 021-1

| θer | Transmission accuracy |
| θ₁ | Input angle |
| θ₂ | Actual output angle |
| R | Reduction ratio |

Formula 021-1

θer = θ₁ - θ₂ / R

Vibration

The primary frequency of the transmission error of the HarmonicDrive® gear may cause a vibration of the load inertia. This can occur when the driving frequency of the servo system including the HarmonicDrive® gear is at, or close to the resonant frequency of the system. Refer to the design guide of each series.

The primary component of the transmission error occurs twice per input revolution of the input. Therefore, the frequency generated by the transmission error is 2x the input frequency (rev/sec).

If the resonant frequency of the entire system, including the HarmonicDrive® gear, is F=15 Hz, then the input speed (N) which would generate that frequency could be calculated with the formula below.

Formula 021-2

N = \frac{15}{2} \cdot 60 = 450 \text{ rpm}

The resonant frequency is generated at an input speed of 450 rpm.

How to calculate resonant frequency of the system

Formula 021-3

f = \frac{1}{2π} \sqrt{ \frac{K}{J} }

Formula variables

Table 021-2

| f | The resonant frequency of the system | Hz |
| K | Spring constant | Nm/rad |
| J | Load inertia | kgm² |

See pages of each series
Starting Torque

Starting torque is the torque value applied to the input side at which the output first starts to rotate. The values in the table of each series indicate the maximum value, and the lower-limit value indicates approximately 1/2 to 1/3 of the maximum value.

**Measurement conditions:**
No-load, ambient temperature: +20°C

- See the corresponding pages of each series for starting torque values.
- Use the values in the table of each series as reference values as they vary depending on the usage conditions.

Backdriving Torque

Backdriving torque is the torque value applied to the output side at which the input first starts to rotate. The values in the table are maximum values, typical values are approximately 1/2 of the maximum values.

**Measurement conditions:**
No-load, ambient temperature: +20°C

- See the corresponding pages of each series for backdriving torque values.
- Use the values in the table of each series as reference values as they vary depending on the usage conditions.

* Contact us for oil lubrication.
No-Load Running Torque

No-load running torque is the torque which is required to rotate the input side (high speed side), when there is no load on the output side (low speed side). The graph of the no-load running torque shown in this catalog depends on the measurement conditions shown in Table 023-1. Add the compensation values shown by each series to all reduction ratios except 100:1.

See the corresponding pages of each series for no-load running torque values.

Efficiency

The efficiency varies depending on the following conditions.
- Reduction ratio
- Input speed
- Load torque
- Temperature
- Lubrication (type and quantity)
The efficiency characteristics of each series shown in this catalog depends on the measurement condition shown in Table 023-2.

See the corresponding pages of each series for efficiency values.

Efficiency compensation coefficient

If load torque is below rated torque, a compensation factor must be employed. Calculate the compensation coefficient Ke from the efficiency compensation coefficient graph of each series and use the following example for calculation.

Example of calculation

Efficiency \( \eta \) (%) under the following condition is obtained from the example of CSF-20-80-2A-GR.
- Input rotational speed: 1000 rpm
- Load torque: 19.6 Nm
- Lubrication method: Grease lubrication (Harmonic Grease SK-1A)
- Lubricant temperature: 20°C
Since the rated torque of size 20 with a reduction ratio of 80 is 34 Nm (Ratings: Page 039), the torque ratio \( \alpha \) is 0.58. 
\[
\alpha = \frac{19.6}{34} = 0.58
\]
- The efficiency compensation coefficient is \( Ke = 0.93 \) from Graph 023-1.
- Efficiency \( \eta \) at load torque 19.6 Nm: \( \eta = Ke \cdot \eta_R = 0.93 \times 78 = 73\% \)
Design Guidelines

**Design guideline**

The relative perpendicularity and concentricity of the three basic Harmonic Drive® elements have an important influence on accuracy and service life.

Misalignments will adversely affect performance and reliability. Compliance with recommended assembly tolerances is essential in order for the advantages of Harmonic Drive® gearing to be fully realized. Please consider the following when designing:

1. Input shaft, Circular Spline and housing must be concentric.

2. When operating, an axial force is generated on the wave generator. Input bearings must be selected to accommodate this axial load. See page 27.

3. Even though a HarmonicDrive® gear is compact, it transmits large torques. Therefore, assure that all required bolts are used to fastened the circular spline and flexspline and that they are tightened to the recommended torque.

4. As the flexspline is subject to elastic deformation, a minimal clearance between the flexspline and housing is required. Refer to “Minimum Housing Clearance” on the drawing dimension tables.

5. The input shaft and output shaft are supported by anti-friction bearings. As the wave generator and flexspline elements are meant to transmit pure torque only, the bearing arrangement needs to isolate the harmonic gearing from external forces applied to either shaft. A common bearing arrangement is depicted in the diagram.

6. A clamping plate is recommended (item 6). Its purpose is to spread fastening forces and to avoid any chance of making physical contact with the thin section of the flexspline diaphragm. The clamping plate shall not exceed the diaphragm’s boss diameter and is to be designed in accordance with catalog recommendations.

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**Fig. 024-1**

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**Fig. 025-1**

---
**Bearing support for the input and output shafts**

For the component sets, both input and output shafts must be supported by two adequately spaced bearings in order to withstand external radial and axial forces without excessive deflection. In order to avoid damage to the component set when limited external loads are anticipated, both input and output shafts must be axially fixed.

Bearing must be selected whose radial play does not exceed ISO-standard C 2 class or “normal” class. The bearings should be axially and radially preloaded to eliminate backlash. Examples of correct bearing arrangements are shown in fig 025-1.

![Fig. 025-1](image-url)
Wave generator

Structure of the wave generator
The wave generator includes an Oldham’s coupling type with a self-aligning structure and an integrated solid wave generator without a self-aligning structure, and which is used depends on the series. See the diagram of each series for details. The basic structure of the wave generator and the shape are shown below.

![Diagram of wave generator components](image)

- Ball Separator
- Wave generator bearing
- Wave generator plug
- Insert
- Rubwasher
- Snap ring
- Wave generator hub

Structure of Oldham’s coupling

![Diagram of Oldham’s coupling](image)

- Oldham’s coupling
- Solid wave generator

Hole diameter of the wave generator with Oldham coupling

Max. hole dia. φV

Min. plug thick. φH

Hole diameter of the wave generator without Oldham Coupling

Maximum hole dim. φV’

Minimum hole dim. φH’

Standard dim. (H7 Dm)

Calculation example

Output torque: 382 Nm
Reduction ratio: 50
Size: 32
Model name: CSF series

Axial force direction of the wave generator

Fig. 027-2

Axial force during acceleration or deceleration

Direction of constant velocity

Calculation formula 027-1

Max. hole diameter of wave generator

<table>
<thead>
<tr>
<th>Size</th>
<th>F</th>
<th>2F</th>
<th>F’</th>
<th>2F’</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>3</td>
<td>-0.1</td>
<td>0</td>
<td>—</td>
</tr>
<tr>
<td>35</td>
<td>5</td>
<td>3</td>
<td>5</td>
<td>—</td>
</tr>
<tr>
<td>40</td>
<td>8</td>
<td>11</td>
<td>14</td>
<td>17</td>
</tr>
<tr>
<td>45</td>
<td>20</td>
<td>25</td>
<td>32</td>
<td>40</td>
</tr>
<tr>
<td>50</td>
<td>50</td>
<td>58</td>
<td>65</td>
<td>80</td>
</tr>
<tr>
<td>55</td>
<td>80</td>
<td>90</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>13</td>
<td>20</td>
<td>25</td>
<td>32</td>
</tr>
<tr>
<td>65</td>
<td>28</td>
<td>35</td>
<td>40</td>
<td>45</td>
</tr>
<tr>
<td>70</td>
<td>47</td>
<td>54</td>
<td>60</td>
<td>65</td>
</tr>
</tbody>
</table>

Output torque: 382 Nm
Reduction ratio: 50
Size: 32
Model name: CSF series

Maximum axial force of the Wave Generator can be calculated by the equation shown below. The axial force acts to push the Wave Generator out of the Flexspline cup.

When the gear is used to accelerate a load, the deflection of the Flexspline, must be supported by the bearings of the input shaft (motor shaft). When the gear is used to decelerate a load, an axial force, which acts in the direction of the closed end of the Wave Generator to the input (motor) shaft. Please contact us for further information on attaching the Wave Generator.
Maximum hole diameter of wave generator

The standard hole dimension of the wave generator is shown for each size. The dimension can be changed within a range up to the maximum hole dimension. We recommend the dimension of keyway based on JIS standard. It is necessary that the dimension of keyways should sustain the transmission torque.

* Tapered holes are also available.

In cases where a larger hole is required, use the wave generator without the Oldham coupling. The maximum diameter of the hole should be considered to prevent deformation of the Wave Generator plug by load torque. The dimension is shown in the table below and includes the dimension of depth of keyway. (This is the value including the dimension of the depth of keyway.)

### Table 027-1

<table>
<thead>
<tr>
<th>Size</th>
<th>8</th>
<th>11</th>
<th>14</th>
<th>17</th>
<th>20</th>
<th>25</th>
<th>32</th>
<th>40</th>
<th>45</th>
<th>50</th>
<th>58</th>
<th>65</th>
<th>80</th>
<th>90</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard dim. (H7)</td>
<td>3</td>
<td>5</td>
<td>6</td>
<td>8</td>
<td>9</td>
<td>11</td>
<td>14</td>
<td>14</td>
<td>19</td>
<td>19</td>
<td>22</td>
<td>24</td>
<td>28</td>
<td>28</td>
<td>28</td>
</tr>
<tr>
<td>Minimum hole dim.</td>
<td>—</td>
<td>—</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>6</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>13</td>
<td>16</td>
<td>16</td>
<td>19</td>
<td>22</td>
</tr>
<tr>
<td>Maximum hole dim.</td>
<td>—</td>
<td>—</td>
<td>8</td>
<td>10</td>
<td>13</td>
<td>15</td>
<td>15</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>25</td>
<td>30</td>
<td>35</td>
<td>37</td>
<td>40</td>
</tr>
</tbody>
</table>

Maximum hole diameter without Oldham Coupling

<table>
<thead>
<tr>
<th>Size</th>
<th>8</th>
<th>11</th>
<th>14</th>
<th>17</th>
<th>20</th>
<th>25</th>
<th>32</th>
<th>40</th>
<th>45</th>
<th>50</th>
<th>58</th>
<th>65</th>
<th>80</th>
<th>90</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max. hole dia.</td>
<td>10</td>
<td>14</td>
<td>17</td>
<td>20</td>
<td>23</td>
<td>28</td>
<td>36</td>
<td>42</td>
<td>47</td>
<td>52</td>
<td>60</td>
<td>67</td>
<td>72</td>
<td>84</td>
<td>95</td>
</tr>
<tr>
<td>Min. plug thick</td>
<td>5.7</td>
<td>6.7</td>
<td>7.2</td>
<td>7.6</td>
<td>11.3</td>
<td>11.3</td>
<td>13.7</td>
<td>15.9</td>
<td>17.8</td>
<td>19</td>
<td>21.4</td>
<td>23.5</td>
<td>28.5</td>
<td>31.3</td>
<td>34.9</td>
</tr>
</tbody>
</table>

Axial Force of Wave Generator

When the gear is used to accelerate a load, the deflection of the Flexspline leads to an axial force acting on the Wave Generator. This axial force, which acts in the direction of the closed end of the Flexspline, must be supported by the bearings of the input shaft (motor shaft). When the gear is used to decelerate a load, an axial force acts to push the Wave Generator out of the Flexspline cup. Maximum axial force of the Wave Generator can be calculated by the equation shown below. The axial force may vary depending on its operating condition. The value of axial force tends to be a larger number when using high torque, extreme low speed and constant operation. The force is calculated (approximately) by the equation. In all cases, the Wave Generator must be axially (in both directions), as well as torsionally, fixed to the input shaft.

(Note)
Please contact us for further information on attaching the Wave Generator to the input (motor) shaft.

### Formula 027-1

<table>
<thead>
<tr>
<th>Reduction ratio</th>
<th>Calculation formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>( F = 2 \times D \times 0.07 \times \tan 32^\circ )</td>
</tr>
<tr>
<td>50</td>
<td>( F = 2 \times D \times 0.07 \times \tan 30^\circ )</td>
</tr>
<tr>
<td>80 or more</td>
<td>( F = 2 \times D \times 0.07 \times \tan 20^\circ )</td>
</tr>
</tbody>
</table>

### Calculation example

Model name: CSF series
Size: 32
Reduction ratio: 50
Output torque: 382 Nm (maximum allowable momentary torque)

\[
F = 2 \times \frac{382}{(32 \times 0.00254)} \times 0.07 \times \tan 30^\circ
\]

\[
F = 380 \text{N}
\]
Assembly Precautions

Sealing

Sealing is needed to maintain the high durability of the gear and prevent grease leakage. Recommended for all mating surfaces, if the o-ring is not used. Flanges provided with o-ring grooves must be sealed when a proper seal cannot be achieved using the o-ring alone.

- Rotating Parts: Oil seal with spring is needed.
- Mating flange: O-ring or seal adhesive is needed.
- Screw hole area: Screws should have a thread lock (LOCTITE® 242 is recommended) or seal adhesive.

(Note) If you use Harmonic Grease 4BNo.2, strict sealing is required.

Assembly precautions

The wave generator is installed after the flexspline and circular spline. If the wave generator is not inserted into the flexspline last, gear teeth scuffing damage or improper eccentric gear mesh may result. Installation resulting in an eccentric tooth mesh (Dedoidal) will cause noise and vibration, and can lead to early failure of the gear. For proper function, the teeth of the flexspline and Circular Spline mesh symmetrically.

- Precautions on the wave generator
  1. Avoid applying undue axial force to the wave generator during installation. Rotating the wave generator bearing while inserting it is recommended and will ease the process.
  2. If the wave generator does not have an Oldham coupling, extra care must be given to ensure that concentricity and inclination are within the specified limits

- Precautions on the circular spline
  The circular Spline must not be deformed in any way during the assembly. It is particularly important that the mounting surfaces are prepared correctly
  1. Mounting surfaces need to have adequate flatness, smoothness, and no distortion.
  2. Especially in the area of the screw holes, burrs or foreign matter should not be present.
  3. Adequate relief in the housing corners is needed to prevent interference with the corner of the circular spline.
  4. The circular spline should be rotatable within the housing. Be sure there is no interference and that it does not catch on anything.
  5. When a bolt is inserted into a bolt hole during installation, make sure that the bolt fits securely and is not in an improper position or inclination.
  6. Do not apply torque at recommended torque all at once. First, apply torque at about half of the recommended value to all bolts, then tighten at recommended torque. Order of tightening bolts must be diagonal.
  7. Avoid pinning the circular spline if possible as it can reduce the rotational precision and smoothness of operation.

- Precautions on the flexspline
  1. Mounting surfaces need to have adequate flatness, smoothness, and no distortion.
  2. Especially in the area of the screw holes, burrs or foreign matter should not be present.
  3. Adequate clearance with the housing is needed to ensure no interference especially with the major axis of flexspline
  4. Bolts should rotate freely when installing through the mounting holes of the flexspline and should not have any irregularity due to the shaft bolt holes being misaligned or oblique.
  5. Do not tighten the bolts with the specified torque all at once. Tighten the bolts temporarily with about half the specified torque, and then tighten them to the specified torque. Tighten them in an even, crisscross pattern.
  6. The flexspline and circular spline are concentric after assembly. After installing the wave generator bearing, if it rotates in unbalanced way, check the mounting for dedoidal or non-concentric installation.
  7. Care should be taken not to damage the flexspline diaphragm or gear teeth during assembly.
  Avoid hitting the tips of the flexpline teeth and circular spline teeth.

- Rust prevention
  Although the Harmonic Drive® gears come with some corrosion protection, the gear can rust if exposed to the environment. The gear external surfaces typically have only a temporary corrosion inhibitor and some oil applied. If an anti-rust product is needed, please contact us to review the options.

Sealing recommendations for gear units

<table>
<thead>
<tr>
<th>Area requiring sealing</th>
<th>Recommended sealing method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Holes which penetrate housing</td>
<td>Use O-ring (supplied with the product)</td>
</tr>
<tr>
<td>Installation screw / bolt</td>
<td>Screw lock adhesive which has effective seal (LOCTITE® 242 is recommended)</td>
</tr>
<tr>
<td>Flange surfaces</td>
<td>Use O-ring (supplied with the product)</td>
</tr>
<tr>
<td>Motor output shaft</td>
<td>Please select a motor which has an oil seal on the output shaft.</td>
</tr>
</tbody>
</table>
"Dedoidal" state

It is normal for the flexspline to engage with the circular spline symmetrically as shown in Figure 029-1. However, if the ratcheting phenomenon, which is described on Page 013, is caused or if the three parts are forcibly inserted and assembled, engagement of the teeth may be out of alignment as shown in Figure 029-2. This is called "dedoidal". Note: Early failure of the gear will occur.

■ How to check "dedoidal"

By performing the following methods, check whether the gear engagement is "dedoidal".

1. Judging by the irregular torque generated when the wave generator turns
   1) Slowly turn the input shaft with your hand in a no-load condition. If you can turn it with average force, it is normal. If it turns irregularly, it may be "dedoidal".
   2) Turn the wave generator in a no-load condition if it is attached to a motor. If the average current value of the motor is about 2 to 3 times the normal value, it may be "dedoidal".

2. Judging by measuring vibration on the body of the flexspline
   The scale deflection of the dial gauge draws a sine wave as shown by the solid line in Graph 029-3 when it is normally assembled. When "dedoidal" occurs, the gauge draws a deflected wave shown by the dotted line as the flexspline is out of alignment.
Checking Output Bearing

A precision cross roller bearing is built in the unit type and the gear head type to directly support the external load (output flange) (precision 4-point contact ball bearing for the CSF-mini series). Please calculate maximum moment load, life of cross roller bearing, and static safety factor to fully maximize the performance of a housed unit (gearhead).

- See the corresponding pages on each series for cross roller bearing specifications.

### Checking procedure

1. **Checking the maximum moment load** \((M_{\text{max}})\)

   - Calculate maximum moment load \((M_{\text{max}})\).
   - Maximum moment load \((M_{\text{max}}) \leq \text{allowable moment}(M_c)\)

2. **Checking the life**

   - Calculate the radial load \((F_{rav})\) and the average axial load \((F_{av})\).
   - Calculate the radial load coefficient \((x)\) and the axial load coefficient \((y)\).
   - Calculate the lifetime.

3. **Checking the static safety coefficient**

   - Calculate the static equivalent radial load coefficient \((P_o)\).
   - Check the static safety coefficient \((f_s)\).

### How to calculate the maximum moment load

Maximum moment load \((M_{\text{max}})\) is obtained as follows. Make sure that \(M_{\text{max}} \leq M_c\).

\[
M_{\text{max}} = F_{rav} (L_r+R) + F_{av} \cdot L_a
\]

### Symbols for Formula 030-1

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>(F_{rav})</td>
<td>Max. radial load</td>
<td>N(kgf)</td>
<td>See Fig. 030-1.</td>
</tr>
<tr>
<td>(F_{av})</td>
<td>Max. axial load</td>
<td>N(kgf)</td>
<td>See Fig. 030-1.</td>
</tr>
<tr>
<td>(L_r, L_a)</td>
<td>—</td>
<td>m</td>
<td>See Fig. 030-1.</td>
</tr>
<tr>
<td>(R)</td>
<td>Offset amount</td>
<td>m</td>
<td>See Fig. 030-1 and “Specification of the output bearing” of each series.</td>
</tr>
</tbody>
</table>
How to calculate the average load

(Average radial load, average axial load, average output speed)

When the radial load and axial load vary, the life of cross roller bearing can be determined by converting to an average load.

How to calculate the average radial load (Frav)

Formula 031-1

\[
Frav = \frac{n_1 t_1 (Fr_{1}) + n_2 t_2 (Fr_{2}) + \cdots + n_n t_n (Fr_{n})}{n_1 t_1 + n_2 t_2 + \cdots + n_n t_n}
\]

(Cross roller bearing)

\[
Frav = \frac{n_1 t_1 (Fr_{1}) + n_2 t_2 (Fr_{2}) + \cdots + n_n t_n (Fr_{n})}{n_1 t_1 + n_2 t_2 + \cdots + n_n t_n}
\]

(4-point contact ball bearing)

Note that the maximum radial load in t is Fr and the maximum radial load in t is Frmax.

How to calculate the average axial load (Faav)

Formula 031-2

\[
Faav = \frac{n_1 t_1 (Fa_{1}) + n_2 t_2 (Fa_{2}) + \cdots + n_n t_n (Fa_{n})}{n_1 t_1 + n_2 t_2 + \cdots + n_n t_n}
\]

(Cross roller bearing)

\[
Faav = \frac{n_1 t_1 (Fa_{1}) + n_2 t_2 (Fa_{2}) + \cdots + n_n t_n (Fa_{n})}{n_1 t_1 + n_2 t_2 + \cdots + n_n t_n}
\]

(4-point contact ball bearing)

Note that the maximum axial load in t is Fa and the maximum axial load in t is Famax.

How to calculate the average output speed (Nav)

Formula 031-3

\[
Nav = \frac{n_1 t_1 + n_2 t_2 + \cdots + n_n t_n}{t_1 + t_2 + \cdots + t_n}
\]

How to calculate the radial load coefficient (X) and axial load coefficient (Y)

Formula 031-4

<table>
<thead>
<tr>
<th>How to calculate the load coefficient</th>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frav / (Frav + 2 (Lr + R) + Faav + La) / dp</td>
<td>&lt;=1.5</td>
<td>0.45</td>
</tr>
<tr>
<td>Frav / (Frav + 2 (Lr + R) + Faav + La) / dp</td>
<td>&gt;1.5</td>
<td>0.67</td>
</tr>
</tbody>
</table>

Symbols for Formula 031-4

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frav</td>
<td>Average radial load</td>
<td>N(kgf)</td>
</tr>
<tr>
<td>Faav</td>
<td>Average axial load</td>
<td>N(kgf)</td>
</tr>
<tr>
<td>Lr, La</td>
<td>Offset amount</td>
<td>m</td>
</tr>
<tr>
<td>R</td>
<td>Pitch circle diameter of a roller</td>
<td>m</td>
</tr>
</tbody>
</table>

For more information, see the corresponding pages on each series for cross roller bearing specifications.
Life of the output bearing

Calculate life of the output bearing by Formula 032-1.
You can calculate the dynamic equivalent radial load (Pc) by Formula 032-2.

**Formula 032-1**

\[
L_{10} = \frac{10^7}{60 \times N_{av}} \times \left( \frac{C}{f_w \times P_c} \right)^{0.3}
\]

**Formula 032-2**

\[
P_c = X \times \left( \frac{F_{av} + \frac{2(F_{av}(L_r+R) + F_{av} \times La)}{dp}}{Y \times F_{av} r} \right)
\]

**Symbols for Formula 032-2**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frav</td>
<td>Average radial load</td>
<td>N (kgf)</td>
</tr>
<tr>
<td>Favar</td>
<td>Average axial load</td>
<td>N (kgf)</td>
</tr>
<tr>
<td>dp</td>
<td>Pitch circle diameter</td>
<td>m</td>
</tr>
<tr>
<td>X</td>
<td>Radial load coefficient</td>
<td>——</td>
</tr>
<tr>
<td>Y</td>
<td>Axial load coefficient</td>
<td>——</td>
</tr>
<tr>
<td>Lr, La</td>
<td>Radial load coefficient</td>
<td>m</td>
</tr>
<tr>
<td>R</td>
<td>Offset</td>
<td>m</td>
</tr>
<tr>
<td>M ave</td>
<td>Average moment load</td>
<td>Nm</td>
</tr>
</tbody>
</table>

**Load coefficient**

<table>
<thead>
<tr>
<th>Load status</th>
<th>fw</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steady operation without impact and vibration</td>
<td>1 to 1.2</td>
</tr>
<tr>
<td>Normal operation</td>
<td>1.2 to 1.5</td>
</tr>
<tr>
<td>Operation with impact and vibration</td>
<td>1.5 to 3</td>
</tr>
</tbody>
</table>
How to calculate life during oscillating motion

Calculate the life of the cross roller bearing during oscillating motion by Formula 033-1.

**Formula 033-1**

(Cross roller bearing)

\[
L_{oc} = \frac{10^6}{60 \times n_1} \times \frac{90}{9} \times \left( \frac{C}{fw \cdot Pc} \right)^{10/3}
\]

(4-point contact ball bearing)

\[
L_{oc} = \frac{10^6}{60 \times n_1} \times \frac{90}{9} \times \left( \frac{C}{fw \cdot Pc} \right)^{3}
\]

### Symbols for Formula 033-1

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loc</td>
<td>Rated life for oscillating motion (hour)</td>
</tr>
<tr>
<td>n1</td>
<td>Round trip oscillation each minute (cpm)</td>
</tr>
<tr>
<td>C</td>
<td>Basic dynamic rated load (N (kgf))</td>
</tr>
<tr>
<td>Pc</td>
<td>Dynamic equivalent radial load (N (kgf))</td>
</tr>
<tr>
<td>fw</td>
<td>Load coefficient</td>
</tr>
<tr>
<td>(\theta)</td>
<td>Oscillating angle /2 (Degree)</td>
</tr>
</tbody>
</table>

### Table 033-1

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loc</td>
<td>Rated life for oscillating motion (hour)</td>
</tr>
<tr>
<td>n1</td>
<td>Round trip oscillation each minute (cpm)</td>
</tr>
<tr>
<td>C</td>
<td>Basic dynamic rated load (N (kgf))</td>
</tr>
<tr>
<td>Pc</td>
<td>Dynamic equivalent radial load (N (kgf))</td>
</tr>
<tr>
<td>fw</td>
<td>Load coefficient</td>
</tr>
<tr>
<td>(\theta)</td>
<td>Oscillating angle /2 (Degree)</td>
</tr>
</tbody>
</table>

(Note) A small angle of oscillation (less than 5 degrees) may cause fretting corrosion to occur since lubrication may not circulate properly. Contact us if this happens.
How to calculate the static safety coefficient

Basic static rated load is an allowable limit for static load, but its limit is determined by usage. In this case, static safety coefficient of the cross roller bearing can be calculated by Formula 034-2.

\[ P_0 = F_{r_{\text{max}}} + \frac{2M_{\text{max}}}{d_p} + 0.44F_{a_{\text{max}}} \]

Table 034-1

<table>
<thead>
<tr>
<th>Co</th>
<th>Basic static rated load</th>
<th>N(kgf)</th>
<th>See &quot;Specification of the output bearing&quot; of each series.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Po</td>
<td>Static equivalent radial load</td>
<td>N(kgf)</td>
<td>See Formula 034-2.</td>
</tr>
</tbody>
</table>

Table 034-2

<table>
<thead>
<tr>
<th>Fr_{max}</th>
<th>Max. radial load</th>
<th>N(kgf)</th>
<th>See &quot;How to calculate the maximum moment load&quot; on Page 030.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fa_{max}</td>
<td>Max. axial load</td>
<td>N(kgf)</td>
<td></td>
</tr>
<tr>
<td>M_{max}</td>
<td>Max. moment load</td>
<td>Nm(kgfm)</td>
<td></td>
</tr>
</tbody>
</table>

Table 034-3

<table>
<thead>
<tr>
<th>Operating condition of the roller bearing</th>
<th>fs</th>
</tr>
</thead>
<tbody>
<tr>
<td>When high rotation precision is required</td>
<td>≥3</td>
</tr>
<tr>
<td>When shock and vibration are expected</td>
<td>≥2</td>
</tr>
<tr>
<td>Under normal operating condition</td>
<td>≥1.5</td>
</tr>
</tbody>
</table>

fs = \frac{Co}{Po}